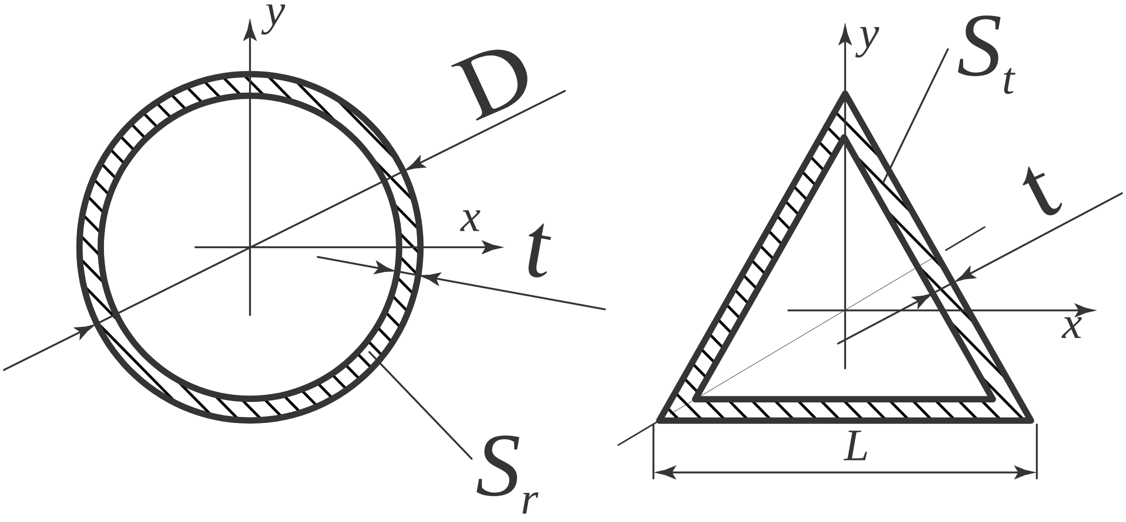
1 Ski pole bending calculation

1.1 Cross-section area



1.1.1 Round tube

D - Outer diameter
t - Wall thickness
S_r - Tube section area
$S_r = \pi \left(\frac{D}{2}\right)^2 - \pi \left(\frac{D-2t}{2}\right)^2$
$S_r = \pi t \left(D - t\right)$

1.1.2 Equilateral triangle

L - Side length
t - Wall thickness
S_t - Triangle section area
$S_t = \frac{L^2 \cos \frac{\pi}{6}}{2} - \frac{\left(L - 2\frac{t}{\tan \frac{\pi}{6}}\right)\left(L \cos \frac{\pi}{6} - t - \frac{t}{\sin \frac{\pi}{6}}\right)}{2}$
$S_t = 3t \left(L - \sqrt{3}t\right)$

1.1.3 Equal areas

If we assume the same conicity, for the same weight we have to design poles with same cross-section area. Thus:
$S_r = S_t$
$\pi t \left(D - t\right) = 3t \left(L - \sqrt{3}t\right)$

Solution is: $L = \sqrt{3}t + \frac{1}{3}\pi D - \frac{1}{3}\pi t$

1.2 Moment of Inertia /Second Moment of Area

1.2.1 Round tube

http://en.wikipedia.org/wiki/List_ of_ area_ moments_ of_ inertia

$I_x = I_y$ - Second moment of area with respect to a horizontal (vertical) axis through the centroid.

In this case, the minimum second moment of area is given as:

$I_{\min}^{\bigcirc} = \frac{\pi}{64}\left(D^4 - (D-2t)^4\right)$

$I_{\min}^{\bigcirc} = -\frac{1}{8}\pi t\left(-D^3 + 3D^2t - 4Dt^2 + 2t^3\right)$

1.2.2 Equilateral triangle

http://www.efunda.com/math/areas/triangle.cfm

I_x - Moment of Inertia about the x axis

I_y - Moment of Inertia about the y axis

For the equilateral triangle, second moment of area is the same for all axes passing through the centroid and is given as:

If x axis and y axis go across the triangle centre, $I_x = I_y$ (centroid)

$I_{\min}^{\triangle} = \frac{L\left(L\cos\frac{\pi}{6}\right)^3}{36} - \frac{\left(L - 2\frac{t}{\tan\frac{\pi}{6}}\right)\left(L\cos\frac{\pi}{6} - t - \frac{t}{\sin\frac{\pi}{6}}\right)^3}{36} = \frac{1}{4}t\left(L^3 - 3\sqrt{3}L^2t + 12Lt^2 - 6\sqrt{3}t^3\right)$

Because $L = \sqrt{3}t + \frac{1}{3}\pi D - \frac{1}{3}\pi t$ (see above), we got:

$I_{\min}^{\triangle} = \frac{1}{108}\pi t\left(D - t\right)\left(\pi^2D^2 + \pi^2t^2 + 27t^2 - 2\pi^2Dt\right)$

1.2.3 Equal-to-or-greater-than?

Let us to take some very realistic values for pole shaft diameter and wall's (both round and triangle shafts) thickness:

$D = 16.0\,\mathrm{mm}$

$t = 1.0\,\mathrm{mm}$

$I_{\min}^{\bigcirc} = \left[\left[-\frac{1}{8}\pi t\left(-D^3 + 3D^2t - 4Dt^2 + 2t^3\right)\right]_{t=1.0}\right]_{D=16.0} = 1331.2\,\mathrm{mm}^4$

$I_{\min}^{\triangle} = \left[\left[\frac{1}{108}\pi t\left(D - t\right)\left(\pi^2D^2 + \pi^2t^2 + 27t^2 - 2\pi^2Dt\right)\right]_{t=1.0}\right]_{D=16.0} = 980.73\,\mathrm{mm}^4$

$\frac{100.0(1331.2 - 980.73)}{980.73} = 35.736\ \%$

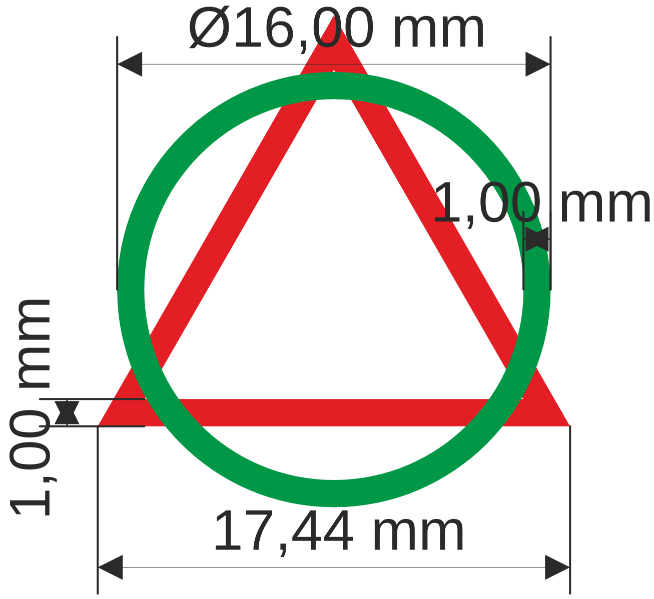
Thus, pole with triangle cross-section is over 35 % more flexible (less rigid) than pole with round cross-section. By this reason our poles are round.

QED

The actual cross-sections Thickness of walls is 1.0 mm

Diameter of round tube is constant 16.0 mm

Outer length of the triangle's side is 17.44 mm

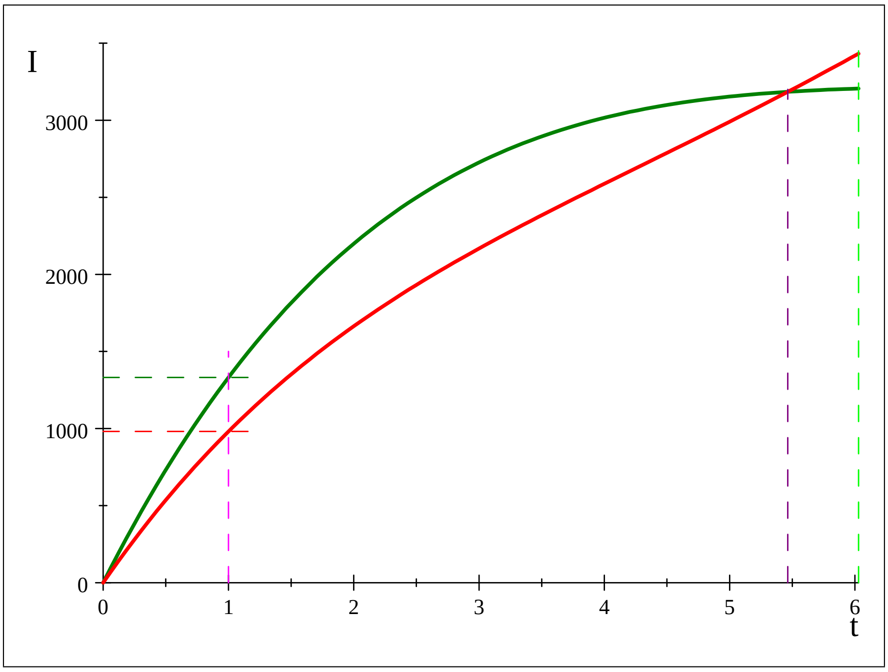


Plot Thickness-Moment of Inertia Thickness of walls is variable (t)

Diameter of round tube is constant 16.0 mm

Outer length of the triangle's side is variable

Maximum possible thickness of triangle's wall is 6.029 mm



Moment of Inertia for equal areas of cross-sections (OD = 16 mm)

1.2.4 Another model

Let us to take some very realistic values for pole shaft diameter and wall's (only round shaft) thickness:

$D = 16.0\,\mathrm{mm}$

$t = 1.0\,\mathrm{mm}$

In this case $S_r = S_t = 47.124\,\mathrm{mm}^2$

Wall's thickness of the triangle shafts is variable. Thus:

$S_t = 3t\left(l - \sqrt{3}t\right) = 47.124$, Solution is: $\frac{0.004}{t}\left(433.01t^2 + 3927.0\right)$

$L = \frac{0.004}{t}\left(433.01t^2 + 3927.0\right)$

$I_{\min}^{\triangle} = \left[\frac{1}{4}t\left(L^3 - 3\sqrt{3}L^2t + 12Lt^2 - 6\sqrt{3}t^3\right)\right]_{L=\left(\frac{0.004}{t}(433.01t^2+3927.0)\right)} =$

$= \frac{1.6\times10^{-14}}{t^2}\left(-5.066\times10^8t^6 + 7.3631\times10^{14}t^4 - 1.25\times10^{11}t^2 + 6.0560\times10^{16}\right)$

To obtain the same value of $I_{\min}^{\triangle}$ as $I_{\min}^{\bigcirc} = 1331.2\,\mathrm{mm}^4$:

$\frac{1.6\times10^{-14}}{t^2}\left(-5.066\times10^8t^6 + 7.3631\times10^{14}t^4 - 1.25\times10^{11}t^2 + 6.0560\times10^{16}\right) = 1331.2$, Solution is: $\{t = 0.85594\}$

$L = \left[\frac{0.004}{t}\left(433.01t^2 + 3927.0\right)\right]_{t=0.85594} = 19.834$

Yes, we got the triangle shaft with wall thickness 0.856 mm as rigid as round shaft with 1.0 mm wall thickness. However, wall thickness 0.856 mm increases risk to get ski pole broken (lower resistance to impact). Let's to see, how rigid will be round shaft with the same thin wall thickness:

$[\pi t\,(D-t)]_{t=0.855\,94} = 0.855\,94\pi\,(D-0.855\,94)$

$0.855\,94\pi\,(D-0.855\,94) = 47.124$, Solution is: $\{D = 18.\,381\}$

$I^{\bigcirc}_{\min} = \left[\left[-\frac{1}{8}\pi t\left(-D^3+3D^2t-4Dt^2+2t^3\right)\right]_{t=0.855\,94}\right]_{D=18.\,381} = 1813.\,5\,\mathrm{mm}^4$

$\frac{100.0(1813.\,5-1331.\,2)}{1331.\,2} = 36.\,23\ \%$

Again we see, pole with triangle cross-section is over 36 % more flexible (less rigid) than pole with round cross-section.

QED